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Correction for measurement error in multilevel analysis using SQP

Bruno Arpino **Diana Zavala**

Department of Political and Social Sciences and Research and Expertise Centre for Survey Methodology (RECSM), Universitat Pompeu Fabra

Motivation

- Obminipresence of measurement errors in empirical social science research.
- Often social scientists use multilevel models.
- Limited literature on correction for measurement errors in multilevel models in the absence of repeated measures/multiple items.
- Use of SQP to get a priori information on measurement error variances.
- Application to ESS-1 data.

Effects of measurement error (ME)

- Well known in single-level models: attenuation of regression coefficients, inflation of residuals variance.
- In multi-level models effects can be more complicated.
- Evidence shows that similarly to single level models, fixed effects are attenuated.
- Effects of ME also on variance components (random intercept, random slopes...).
- Limited evidence: within/between effects; differential measurement errors (heteroschedastic measurement errors).

Correction for measurement error

- Tipically requires repeated measures on the same subjects or multiple items for the same concept → SEM, test-retest, calibration, etc.
- In many cases, social scientists have to use only one observation per subject and one single item.
- Browne et al (2001) and Goldstein et al (2008) propose to use a MCMC approch to implement a Bayesian classical measurement error model.

The model of interest

$$y_{ij} = X_{ij}\beta + Z_{ij}u_j + e_{ij},$$
$$X_{ij} = (x_{1ij}, x_{2ij}, \ldots, x_{pij}), \quad Z_{ij} = (z_{1ij}, z_{2ij}, \ldots, z_{qij}), \quad u_j^T = (u_{1j}, u_{2j}, \ldots, u_{qj}),$$
$$u_j \sim MVN(0, \Omega_u), \quad e_{ij} \sim N(0, \sigma_e^2),$$

- In a Bayesian framework to make inference about the unkwown parameters we need to find the joint posterior distribution:

$$p(\beta, u, \sigma_u^2, \sigma_e^2 \mid y) \propto p(y \mid \beta, u, \sigma_e^2) p(u \mid \sigma_u^2) p(\beta) p(\sigma_u^2) p(\sigma_e^2).$$

- We can use Monte Carlo Markov Chain (MCMC) procedures to estimate the conditional posteriors.

Gibbs sampling

- First we choose starting values (for example using ML estimation), then we sample from:

$$p(\beta \mid y, u_{(0)}, \sigma^2_{u(0)}, \sigma^2_{e(0)})$$

to get $\beta_{(1)}$, then

$$p(u \mid y, \beta_{(1)}, \sigma^2_{(0)}, \sigma^2_{e(0)})$$

to get $u_{(1)}$, then

$$p(\sigma^2_u \mid y, \beta_{(1)}, u_{(1)}, \sigma^2_{e(0)})$$

to get $\sigma^2_{u(1)}$, then finally

$$p(\sigma^2_e \mid y, \beta_{(1)}, u_{(1)}, \sigma^2_{u(1)})$$

Bayesian classical measurement error model

- Classical measurement error (ME) model:

$$x^0 = x + m \qquad R = R(x^0) = \sigma_x^2 / \sigma_{x^0}^2$$

- If X_1 denotes the matrix of covariates affected by ME:

$$y_{ij} = [X_{1ij}(\beta_1 + Z_{1ij} \cdot U_{1j})] + [X_{2ij}(\beta_2 + Z_{2ij} \cdot U_{2j})] + e_{ij},$$
$$\beta^T = \{\beta_1^T, \beta_2^T\}, \quad Z^T = \{Z_1^T, Z_2^T\}, \quad U = \{U_1, U_2\},$$

Bayesian classical measurement error model

- An extra step is added to the MCMC algorithm for sampling for each row of X_1:

$$X_{1ij} \sim MVN\,(\hat{X}_{1ij}, \hat{V}_{ij}),$$

$$\hat{V}_{ij} = \left[\frac{(\beta_1 + Z_1 \cdot U_{1j})(\beta_1 + Z_1 \cdot U_{1j})^T}{\sigma_e^2} + \Omega_m^{-1} + \Omega_\phi^{-1} \right]^{-1},$$

$$\hat{X}_{1ij} = \hat{V}_{ij} \left[\frac{(\beta_1 + Z_1 \cdot U_{1j})(y_{ij} - X_{2ij}(\beta_2 + Z_2 \cdot U_{2j}))}{\sigma_e^2} + X_{1ij}^0 \Omega_m^{-1} + \theta \Omega_\phi^{-1} \right]$$

SQP as provider of a priori information on ME variances

- In this approach Ω_m is assumed to be known. Goldstein et al (2008) suggest to use increasingly higher values of the variance of the measurment errors in a sort of sensitivity analysis.

- SQP can be used to obtain a priori information on the reliability of the measured variables and so on the variances of the measurement errors.

The application

- We consider the model estimated by Semyonov et al (2008) to analyze the factors affecting attitudes toward immigrants using ESS-1 data (21 European countries).

- Dependent variable: attitudes toward immigrants. Mean of six 0–10 scale items: 0 positive, 10 negative. For example:

  Would you say it is generally bad or good for [country]'s economy that people come to live here from other countries?

The application

- We focus on two independent variables:
  - political orientation ("lrscale"):
    How would you place your views on this scale. 0 left, 10 right.
  - perceived size of immigrant population ("perceived"):
    Out of every 100 people living in [country], how many do you think were born outside [country]?

- Other independent variables: age, gender, education, unemployment status, gdp, % right-wing vote, etc.
- 2-level linear random slopes model.

The application

- We first consider a random intercept model, then we allow for random slopes on perceived size, lrscale, education and unemployment status.
- Weekly informative on random effects variance -covariance matrix.
- Gibbs sampling with a burn-in of 500 with a sample of 5000 iterations.

Quality predictions for the independent variables of interest – perceived size

Country	Language	Quality coefficient	Interquartile rage	Standard error
Belgium	French	0.67	(0.430, 0.784)	0.22
Switzerland	French	0.67	(0.430, 0.771)	0.21
Switzerland	German	0.67	(0.425, 0.776)	0.22
France	French	0.70	(0.444, 0.825)	0.22
Spain	Spanish	0.72	(0.435, 0.842)	0.24
Greece	Greek	0.77	(0.422, 0.888)	0.26
Denmark	Danish	0.80	(0.475, 0.893)	0.23
Portugal	Portuguese	0.83	(0.479, 0.908)	0.24
Austria	German	0.84	(0.480, 0.916)	0.24
Ireland	English	0.84	(0.459, 0.912)	0.25
…….				
Mean		**0.73**		

Quality predictions for the independent variables of interest - left right scale

Country	Language	Quality coefficient	Interquartile rage	Standard error
Germany	German	0.80	(0.671, 0.837)	0.12
UK	English	0.78	(0.655, 0.807)	0.11
Switzerland	Italian	0.76	(0.628, 0.790)	0.12
Netherlands	Dutch	0.85	(0.710, 0.880)	0.11
Czech Republic	Czech	0.75	(0.623, 0.782)	0.12
Spain	Spanish	0.78	(0.665, 0.810)	0.11
France	French	0.74	(0.616, 0.769)	0.11
…...	….....			
Mean		0.78		

Random intercept models: fixed effects

	NO	pe L	pe H	lr L	lr H	pe L & lr L	pe L & lr H	pe H & lr L	pe H & lr H
perceived_CM	0.0114***	0.0126***	0.0190***	0.0114***	0.0114***	0.0127***	0.0127***	0.0191***	0.0191***
	(0.0007)	(0.0007)	(0.0011)	(0.0007)	(0.0007)	(0.0007)	(0.0007)	(0.0011)	(0.0011)
perceivedCM	0.0260	0.0273	0.0273	0.0273	0.0273	0.0227	0.0227	0.0227	0.0227
	(0.0154)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0152)	(0.0152)	(0.0152)	(0.0152)
lrscale_CM	0.0931***	0.0932***	0.0932***	0.1036***	0.1554***	0.1036***	0.1556***	0.1036***	0.1555***
	(0.0047)	(0.0046)	(0.0046)	(0.0052)	(0.0078)	(0.0052)	(0.0077)	(0.0052)	(0.0077)
lrscaleCM	0.8126**	0.9054*	0.9054*	0.9053*	0.9054*	0.8509***	0.8509***	0.8511***	0.8511***
	(0.2936)	(0.3531)	(0.3534)	(0.3532)	(0.3534)	(0.2495)	(0.2493)	(0.2493)	(0.2492)

- Maximal % changes:
  perceived (+68%), lrscale (+67%), perceivedCM (-13%), lrscaleCM (+11%)

Random intercept models: random part

	NO	pe L	pe H	lr L	lr H	pe L & lr L	pe L & lr H	pe H & lr L	pe H & lr H
var(u0)	0.1317*	0.1425*	0.1425*	0.1425*	0.1425*	0.1301*	0.1302*	0.1301*	0.1302*
	(0.0538)	(0.0623)	(0.0623)	(0.0623)	(0.0623)	(0.0536)	(0.0536)	(0.0536)	(0.0536)
var(e)	2.1633***	2.1599***	2.1442***	2.1590***	2.1383***	2.1557***	2.1349***	2.1396***	2.1189***
	(0.0200)	(0.0198)	(0.0198)	(0.0198)	(0.0199)	(0.0199)	(0.0200)	(0.0200)	(0.0201)
ICC	5.7386	6.1892	6.2317	6.1916	6.2478	5.6917	5.7481	5.7320	5.7890

- Maximal % changes:
  var(u0) (+8%), var(e) (-2%), ICC (+9%)

Random slopes models: fixed effects

	NO	pe L	pe H	lr L	lr H	pe L & lr L	pe L & lr H	pe H & lr L	pe H & lr H
perceiv_CM	0.0101***	0.0115**	0.1721	0.0101**	0.0101**	0.0112***	0.0112***	0.0203	0.0193*
	(0.0030)	(0.0038)	(0.1241)	(0.0033)	(0.0033)	(0.0031)	(0.0031)	(0.0123)	(0.0093)
perceivCM	0.0306	0.0204	0.0204	0.0205	0.0205	0.0298	0.0299	0.0299	0.0300
	(0.0176)	(0.0157)	(0.0158)	(0.0157)	(0.0157)	(0.0195)	(0.0196)	(0.0196)	(0.0197)
lrscale_CM	0.0809***	0.0823***	0.0823***	0.0917***	0.1404***	0.0917***	0.1400***	0.0917***	0.1397***
	(0.0167)	(0.0193)	(0.0193)	(0.0218)	(0.0363)	(0.0191)	(0.0288)	(0.0191)	(0.0287)
lrscaleCM	0.9550**	0.8185**	0.8217**	0.8186**	0.8192**	0.9082***	0.9088***	0.9127***	0.9132***
	(0.3589)	(0.3001)	(0.3012)	(0.3006)	(0.3007)	(0.2553)	(0.2548)	(0.2545)	(0.2539)

- Maximal % changes:
  perceived (+101%), lrscale (+74%), perceivedCM (-33%), lrscaleCM (-14%)

Random slopes models: random part

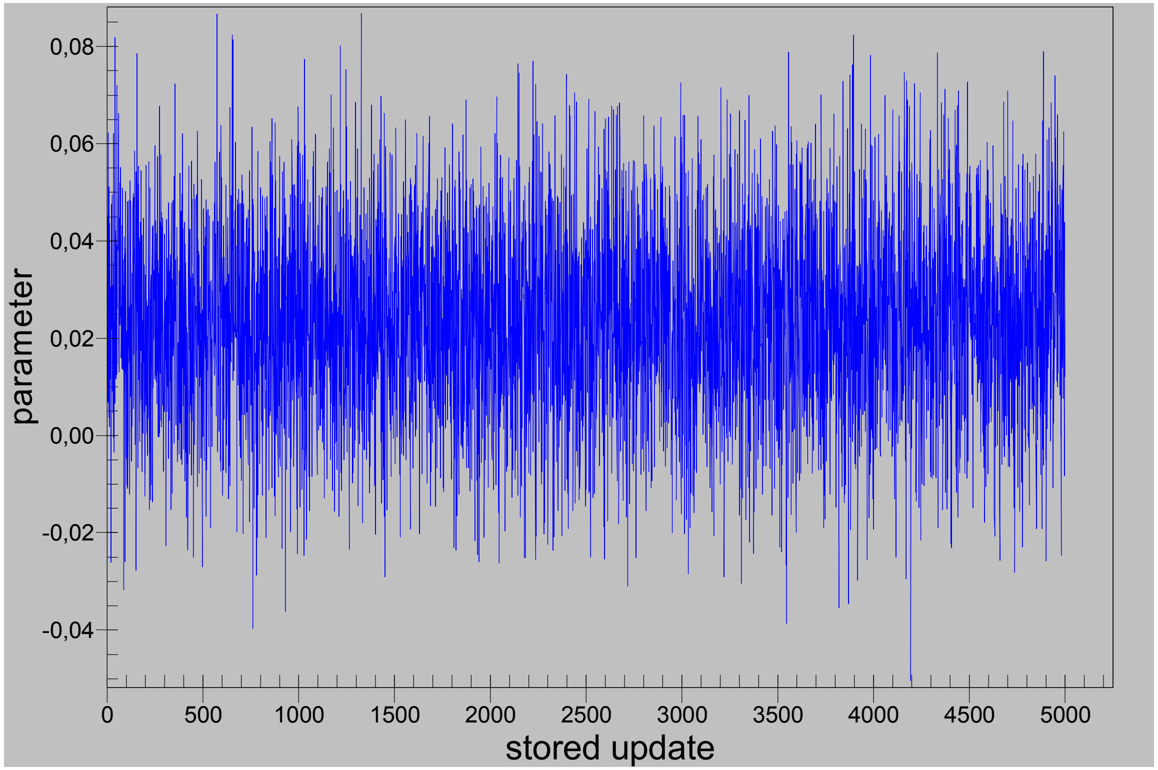
	NO	pe L	pe H	lr L	lr H	pe L & lr L	pe L & lr H	pe H & lr L	pe H & lr H
var(u0)	0.1406*	0.1337*	0.1352*	0.1337*	0.1339*	0.1421*	0.1426*	0.1429*	0.1434*
	(0.0649)	(0.0617)	(0.0621)	(0.0617)	(0.0617)	(0.0718)	(0.0722)	(0.0721)	(0.0724)
var(lrscale_CM)	0.0063**	0.0065*	0.0064*	0.0081*	0.0199*	0.0079**	0.0192**	0.0079**	0.0190**
	(0.0023)	(0.0025)	(0.0025)	(0.0032)	(0.0080)	(0.0030)	(0.0073)	(0.0030)	(0.0073)
var(perceiv_CM)	0.0002**	0.0002*	0.0435	0.0002**	0.0002**	0.0002**	0.0002**	0.0005	0.0004
	(0.0001)	(0.0001)	(0.0445)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0009)	(0.0006)
var(e)	2.1242***	2.1194***	2.0901***	2.1167***	2.0784***	2.1126***	2.0747***	2.0850***	2.0484***
	(0.0194)	(0.0197)	(0.0200)	(0.0197)	(0.0200)	(0.0194)	(0.0197)	(0.0198)	(0.0200)

- Maximal % changes:
  Var(u0) (+2%), var(lrscale) (+202%), var(perceived) (+150%), var(e) (-2%)

Concluding remarks

- Measurement errors in multilevel models have effects that are more complex than in single-level models:
  - Attenuation bias for within effects
  - Inflation of first level error variance and attenuation of random effects (intercepts and slopes) variances
- Applications can benefit from this approach as a robustness check.
- We plan to extend the analysis allowing for: correlations between the measurement errors, heteroschedastic measurement errors, Y with errors.


Thank you!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