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Explaining cross-national inequivalence
A Bayesian Multilevel SEM approach

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Introduction

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- Challenges in measurement equivalence testing
  - Large-scale surveys: increasing number of contexts
  - What to do if equivalence cannot be established?

- Outline:
  - MLSEM approach to measurement equivalence
    - The two-level CFA model
    - Measurement equivalence
    - Bayesian estimation
  - Illustration: citizenship conceptions in ISSP
    - Explaining random slope variation
    - Simulation study

Two-level CFA model

- A multilevel approach to CFA (Muthen 1994)
  - Starting point: population of individuals *i* divided into *g* groups
  - Decomposition of individual data into within group and between group components:

$$X_{ig} = X_W + X_B$$
$$X_{ig} = (X_{ig} - \overline{X_g}) + \overline{X_g}$$

  - Orthogonal decomposition of total covariance structure into within- and between-group covariance structures:

$$\Sigma_T = \Sigma_W + \Sigma_B$$

Two-level CFA model

- One model that simultaneously predicts within- and between-group components of the data:

$$\mathbf{X}_{\mathbf{ig}} = \boldsymbol{\alpha}_{\mathbf{g}} + \boldsymbol{\Lambda}_{\mathbf{W}}\boldsymbol{\eta}_{\mathbf{W}} + \boldsymbol{\delta}_{\mathbf{Wig}} \qquad (1)$$

$$\boldsymbol{\alpha}_{\mathbf{g}} = \boldsymbol{\nu} + \boldsymbol{\Lambda}_{\mathbf{B}}\boldsymbol{\eta}_{\mathbf{B}} + \boldsymbol{\delta}_{\mathbf{Bg}} \qquad (2)$$

- Substition of (2) into (1)

$$\mathbf{X}_{\mathbf{ig}} = \boldsymbol{\nu} + \boldsymbol{\Lambda}_{\mathbf{W}}\boldsymbol{\eta}_{\mathbf{W}} + \boldsymbol{\Lambda}_{\mathbf{B}}\boldsymbol{\eta}_{\mathbf{B}} + \boldsymbol{\delta}_{\mathbf{Bg}} + \boldsymbol{\delta}_{\mathbf{Wig}}$$

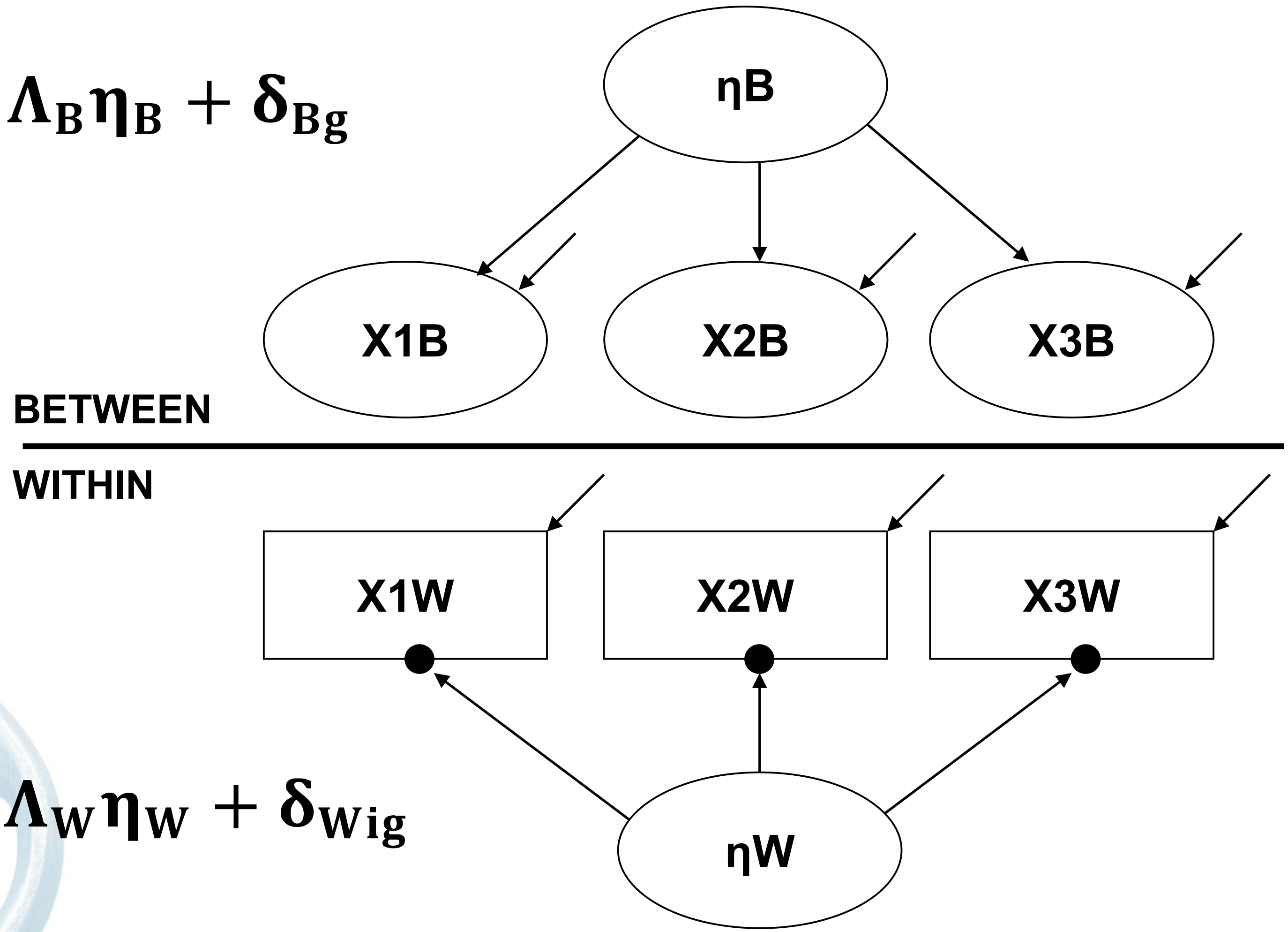
- This model implies:

$$\boldsymbol{\Sigma}_{\mathbf{W}} = \boldsymbol{\Lambda}_{\mathbf{W}}\boldsymbol{\Phi}_{\mathbf{W}}\boldsymbol{\Lambda}'_{\mathbf{W}} + \boldsymbol{\Theta}_{\mathbf{W}}$$

$$\boldsymbol{\Sigma}_{\mathbf{B}} = \boldsymbol{\Lambda}_{\mathbf{B}}\boldsymbol{\Phi}_{\mathbf{B}}\boldsymbol{\Lambda}'_{\mathbf{B}} + \boldsymbol{\Theta}_{\mathbf{B}}$$

Two-level CFA model

$$\alpha_g = \nu + \Lambda_B \eta_B + \delta_{Bg}$$

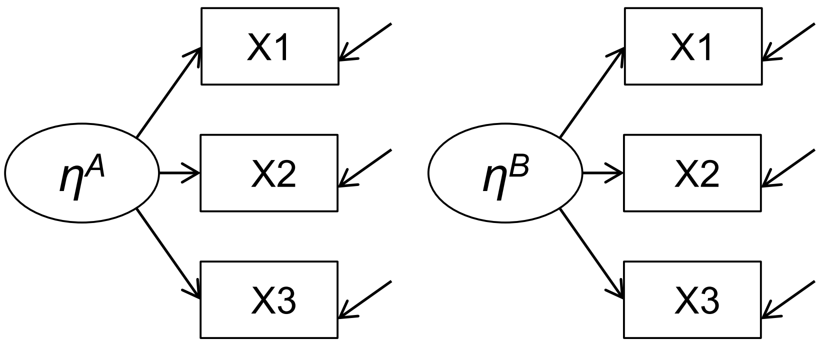


$$X_{ig} = \alpha_g + \Lambda_W \eta_W + \delta_{Wig}$$

$$X_{ig} = \nu + \Lambda_W \eta_W + \Lambda_B \eta_B + \delta_{Bg} + \delta_{Wig}$$

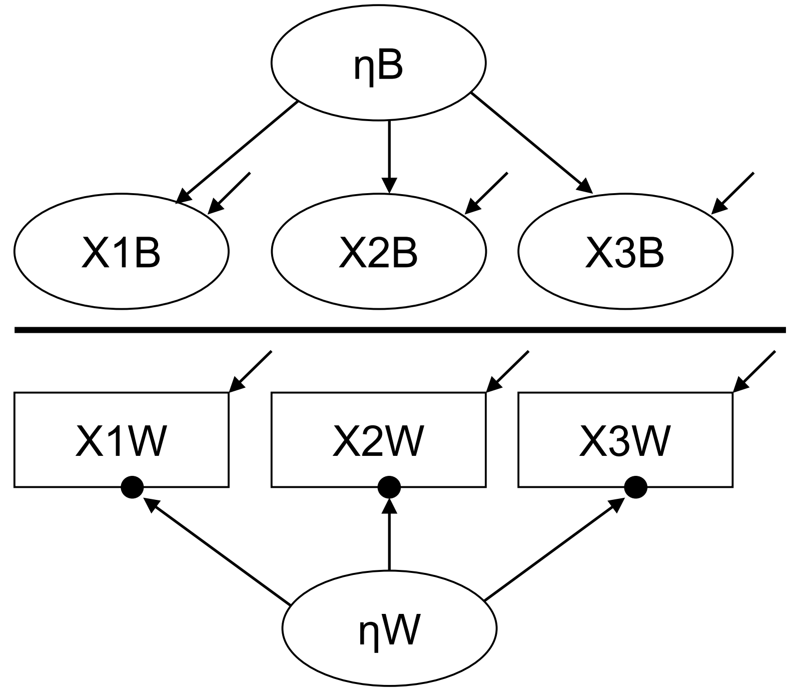
Configural equivalence

Multigroup CFA vs. Multilevel CFA



Group A   Group B

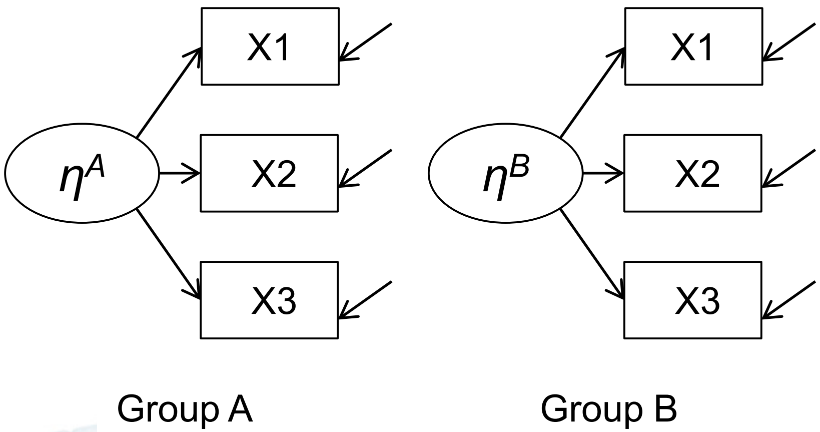
- Equal factor structures across groups



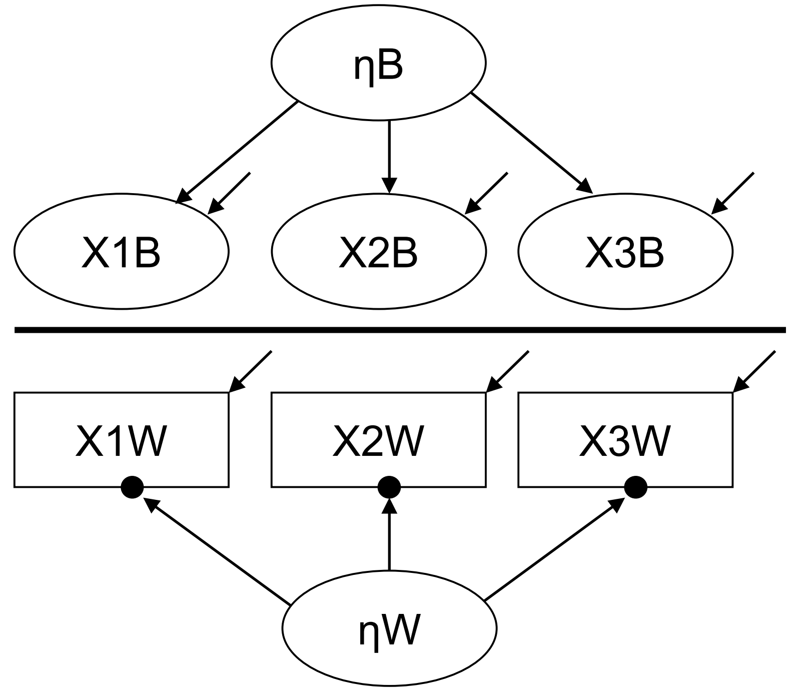
- Equal structures of group-specific within matrices
- Analysis of pooled within matrix → implied by the model

Metric equivalence

Multigroup CFA vs. Multilevel CFA



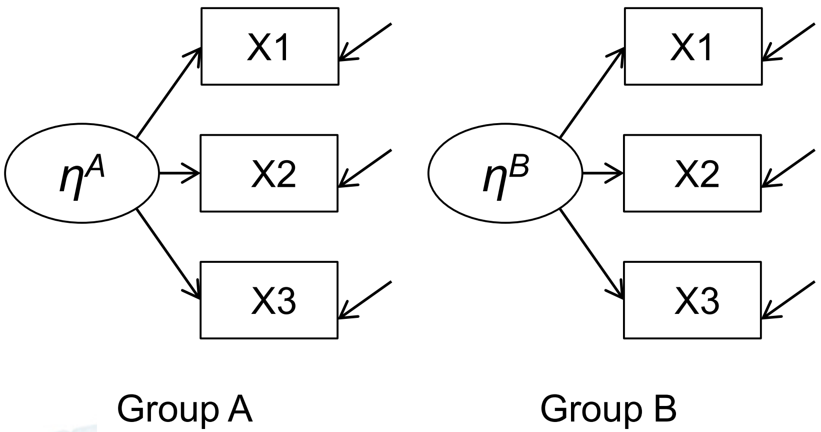
- Equal factor loadings across groups

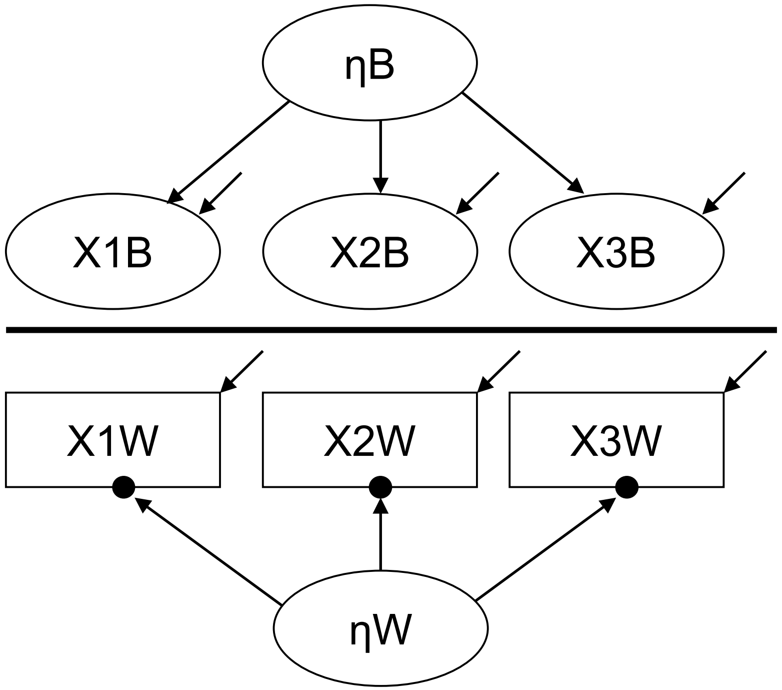


- Single set of factor loadings for pooled within matrix
  → implied by the model
- But: can be overruled by including a random factor loading

Scalar equivalence

Multigroup CFA vs. Multilevel CFA

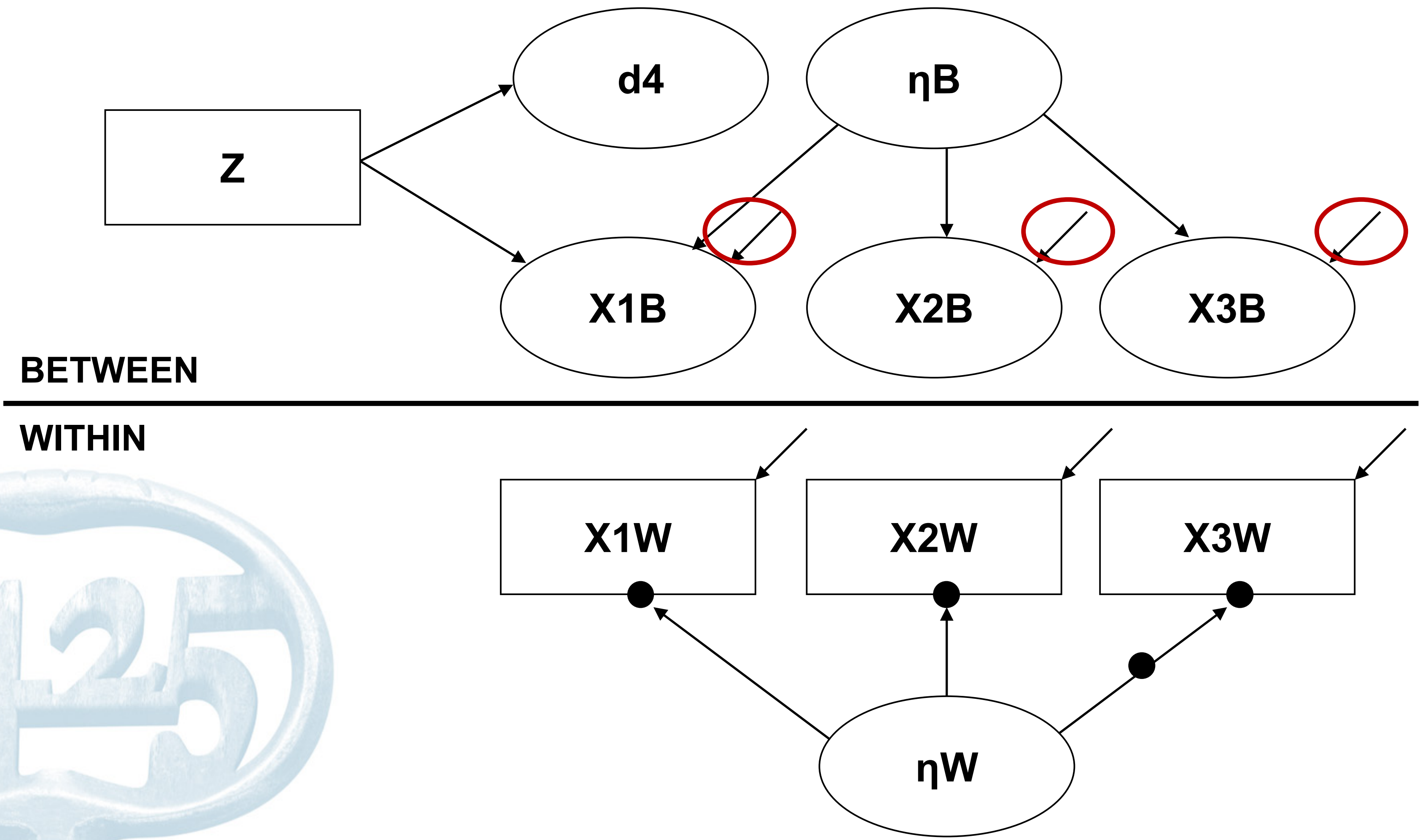




- Equal intercepts across groups

- Between-level residuals contain variations in item means not captured by the latent variable
- To test scalar equivalence, constrain between-level residuals to 0

Explaining inequivalence



Bayesian estimation

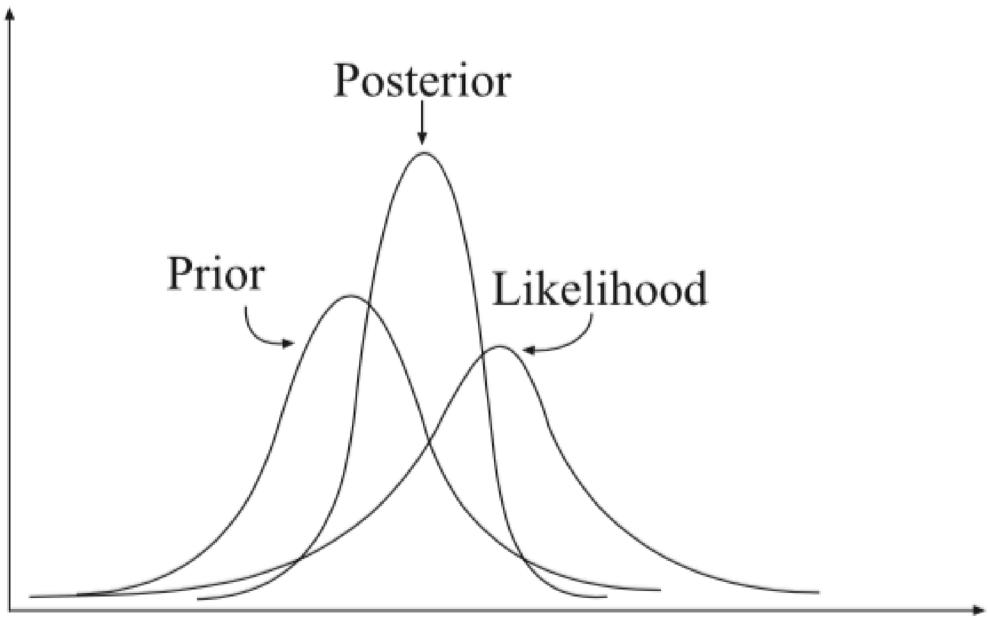
- Bayes' theorem:

P(hypothesis | data) ∝ P(data | hypothesis) x P(hypothesis)

Posterior probability

Likelihood

Prior probability



Advantages:

1. More can be learned from parameters
2. **Better small-sample performance**
3. Computationally less demanding
4. New types of models can be analyzed

Application: citizenship conceptions

- Explaining slope differences in MLSEM:
  - International Social Survey Program
  - 2003 wave: focus on **national identity**
  - 32 societies

Application: citizenship conceptions

- Scale measuring **ethnic citizenship conceptions**:

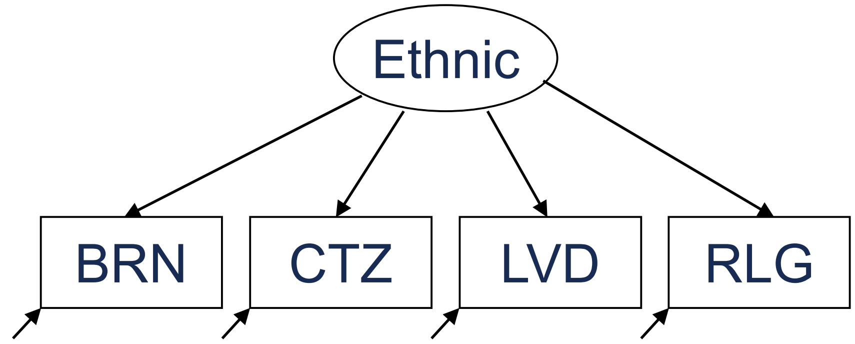
  Some people say that the following things are important for being truly [NATIONALITY]. Others say they are not important. How important do you think each of the following is?

  - To have been born in the country (**BRN**)
  - To have [COUNTRY NATIONALITY] citizenship (**CTZ**)
  - To have lived in [COUNTRY] for most of one's life (**LVD**)
  - To be a [RELIGION] (**RLG**)

- *Answer scale: very important (1) – not important at all (4) (reversed)*

Application: citizenship conceptions

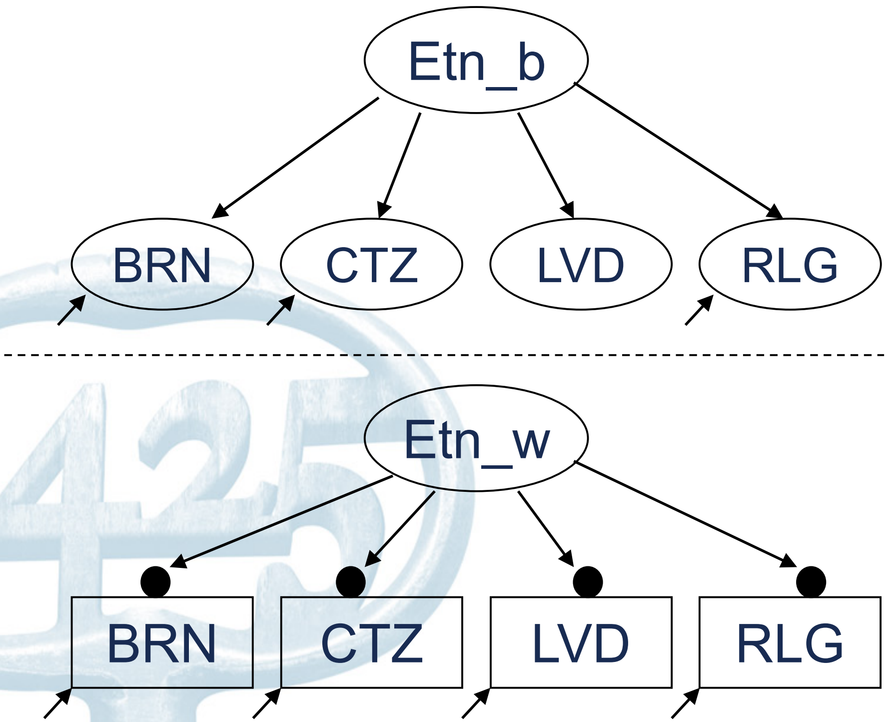
- MGCFA for 32 groups:



	Chi2	Df	RMSEA	CFI	TLI	SRMR
Configural equivalence	359.658	64	0.063	0.992	0.976	0.018
Metric equivalence	2440.424	157	0.112	0.939	0.926	0.145
Scalar equivalance	10850.897	250	0.191	0.718	0.783	0.244

Application: citizenship c

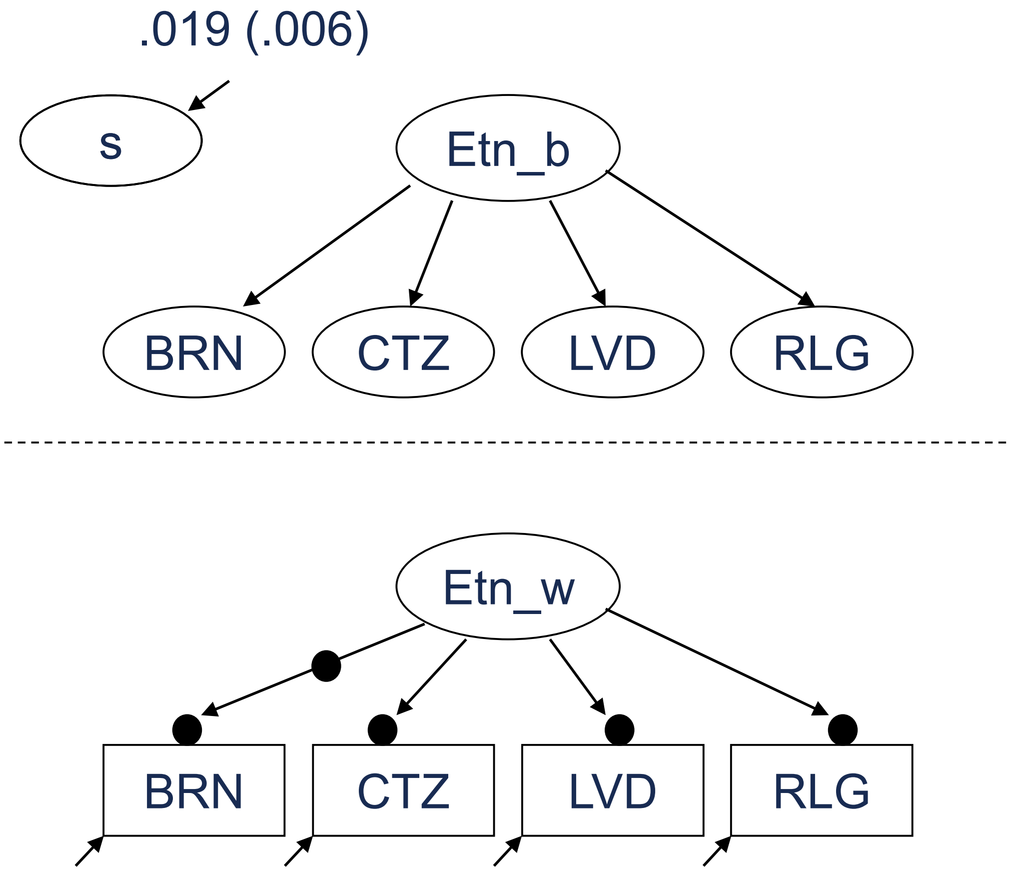
- Two-level CFA:
  - Bayesian estimation
    - Uninformative priors
    - 10.000 iterations



WITHIN MODEL	
Factor loadings	
BRN	0.787 *
CTZ	0.693 *
LVD	0.689 *
RLG	0.426 *
Residual Variances	
BRN	0.381 *
CTZ	0.52 *
LVD	0.526 *
RLG	0.819 *
BETWEEN MODEL	
Factor loadings	
BRN	0.973 *
CTZ	0.694 *
LVD	0.962 *
RLG	0.747 *
Residual Variances	
BRN	0.053 *
CTZ	0.518 *
LVD	0.075 *
RLG	0.442 *

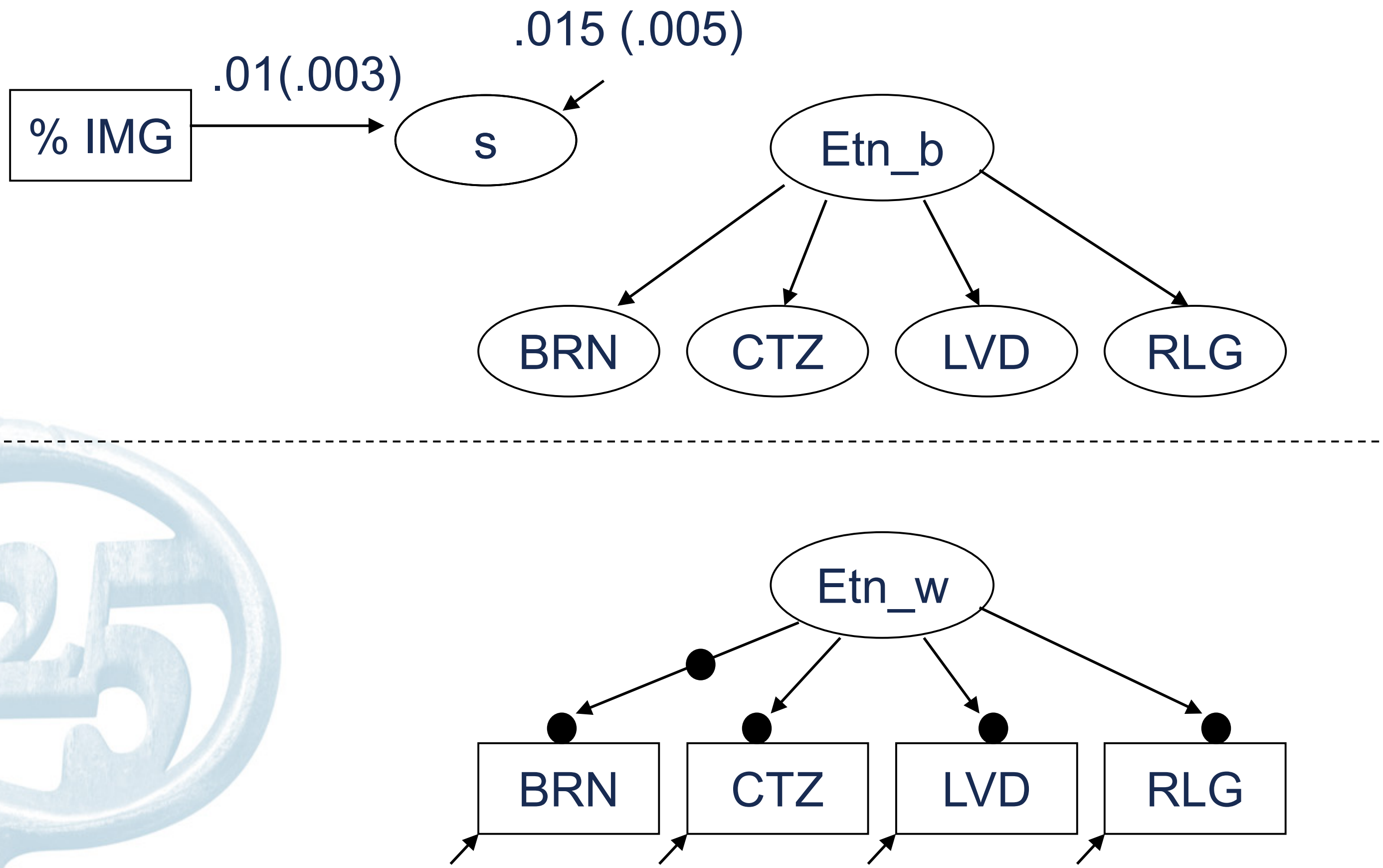
Application: citizenship conceptions

- Set factor loading for BRN random:

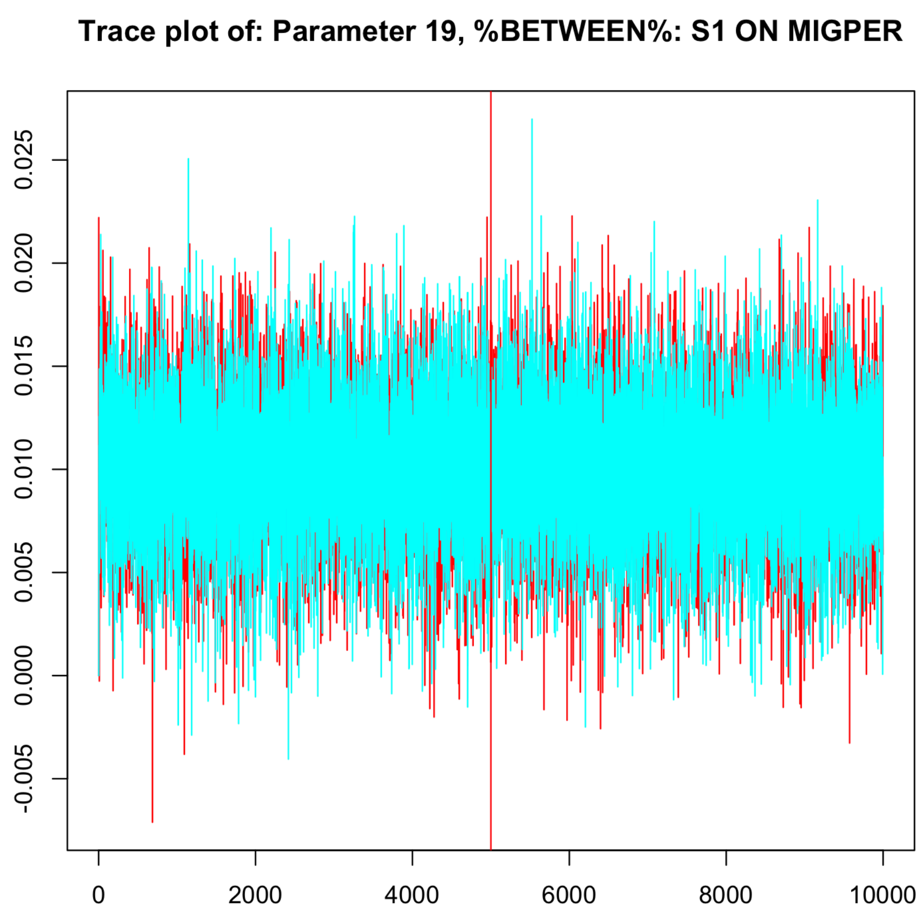


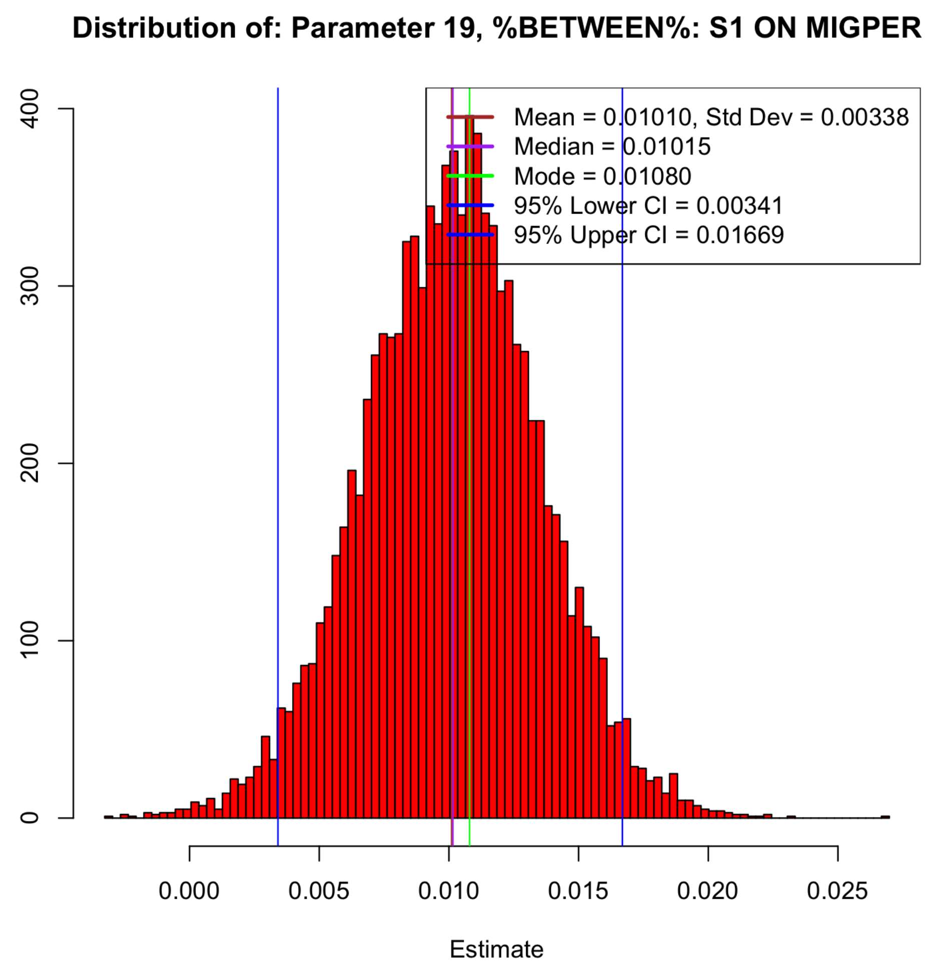
Application: citizenship conceptions

- Effect of % of immigrants (UNDP estimate) on slope variance:



Application: citizenship conceptions





Application: citizenship conceptions

- Monte Carlo simulation (500 replications)

	WITHIN			BETWEEN		
	Parameter bias	***SE bias***	***Coverage***	***Parameter bias***	***SE bias***	***Coverage***
Factor loadings						
Item1	0.03%	1.89%	0.946	6.83%	21.66%	0.952
Item2	-0.03%	1.89%	0.958	7.20%	12.11%	0.938
Item3	-0.03%	0.00%	0.95	6.50%	14.17%	0.932
Item 4	0.08%	0.00%	0.956	6.83%	13.16%	0.938
Residual variances						
Item1	0.04%	2.00%	0.952	13.24%	25.00%	0.952
Item2	-0.02%	-1.92%	0.946	11.21%	20.51%	0.952
Item3	-0.02%	-3.77%	0.938	12.04%	18.99%	0.952
Item4	0.04%	-5.66%	0.946	14.41%	14.29%	0.928
Random slope				7.98%	14.89%	0.946
Effect of Z				-0.14%	9.01%	0.950

Conclusion

- In some cases, measurement inequivalence can be a source of information rather than just nuisance
- MLSEM can help to explain why slopes (and intercepts) vary across groups
- It might even be possible to explain inequivalence away
- Bayesian estimation produces sufficiently reliable inference, even with small sample sizes

- Unresolved issues:
  - Model fit indices in Bayesian MLSEM?
  - Set free multiple factor loadings (cfr. IRT approach)?
  - Small-variance priors on the factor loading variances?

Thank you for your attention!

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