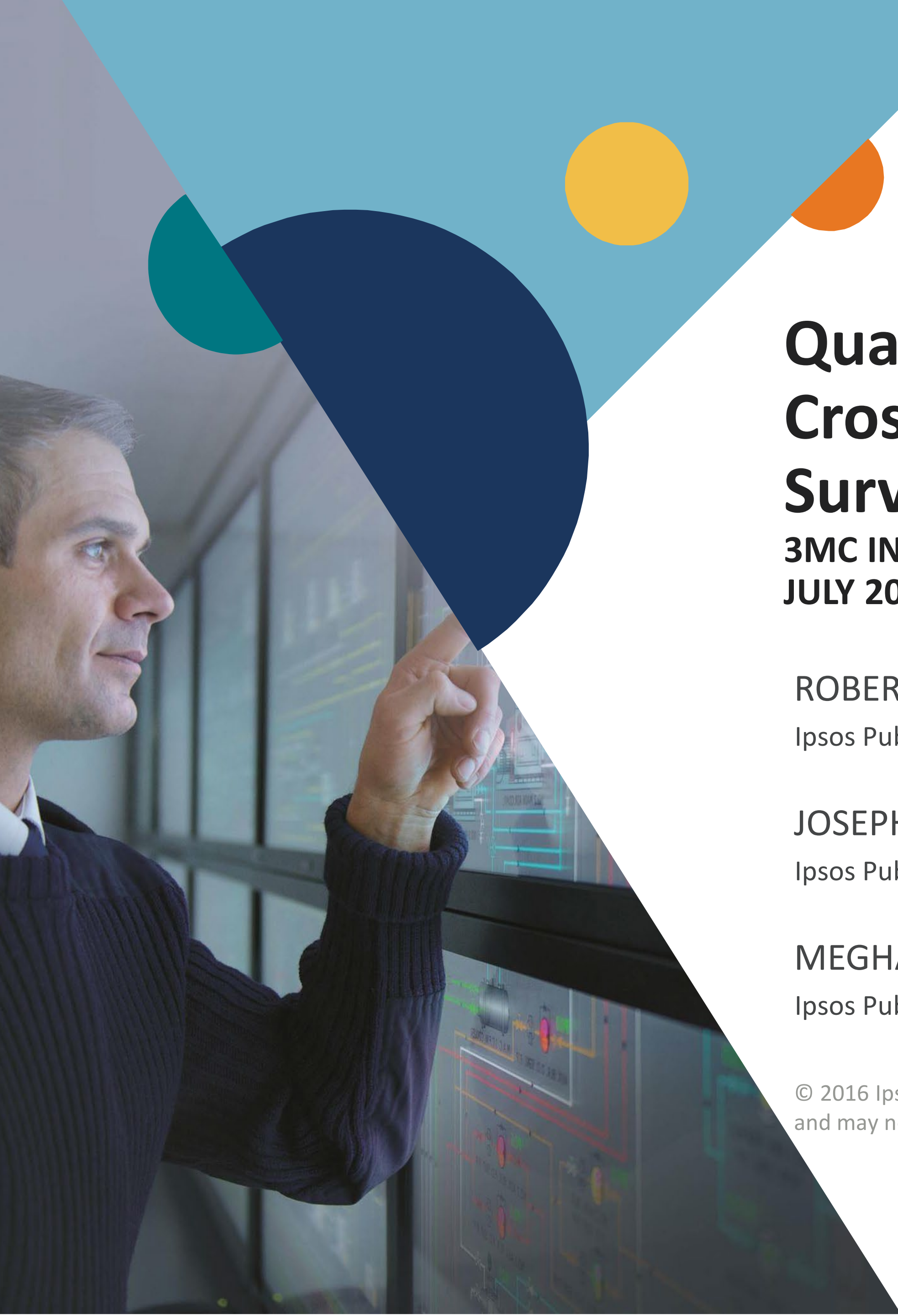

Ipsos Public Affairs

Quantile Regression as a Tool for Cross-National and Comparative Survey Research

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GAME CHANGERS

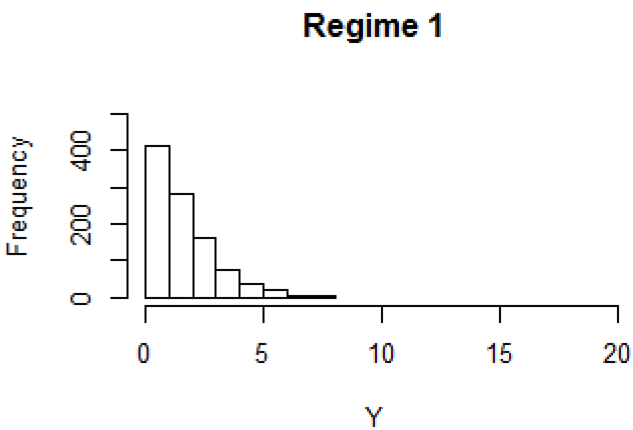
Contents

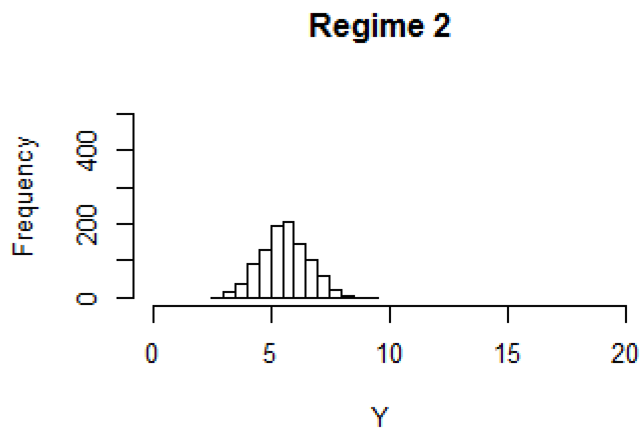
CONTEXT AND MOTIVATING CHALLENGES

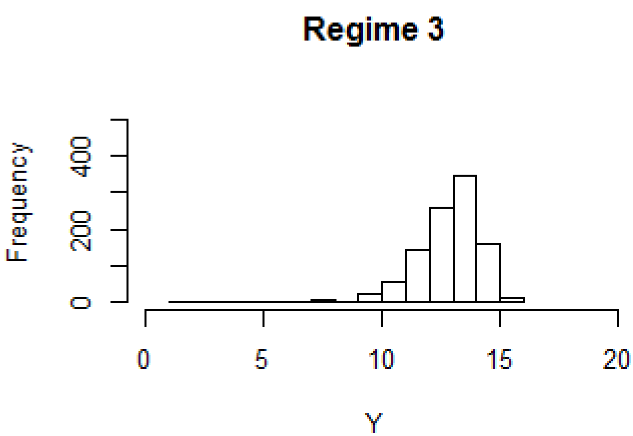
Qualitative and Quantitative Variation

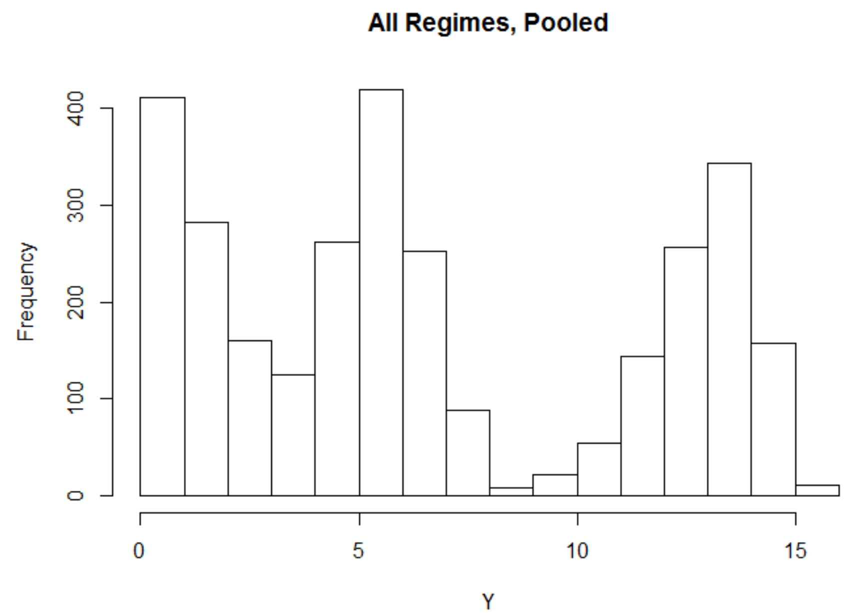
- Survey and evaluation research require researchers to take into account the particulars of populations and phenomena studied
  - **In agricultural surveys**, can often see qualitative variation in key measures across regions and countries, especially when subset on key groups
    - Distribution of arable soil can impact crop yields and crop quality dramatically across study regions
  - **In longitudinal evaluation research**, it is reasonable to expect that the intervention itself can alter both the qualitative and quantitative nature of the focal measures
    - **A business intervention which promotes shifting product mixes** to enhance profitability could lead to temporary re-alignment of goods and services sold and a thus temporary or longer-term shifts in the distribution of sales / profits
    - **An intervention which promotes financial record keeping** could lead to qualitative shifts in the distributions of reported income

Quantitative vs. Qualitative Variation









Other Considerations

- When setting up (parameterizing) quantitative analyses, have to be careful to be context-sensitive and not impose prior conceptions on what population phenomena look like
  - US overall income distribution (gamma-distributed) vs. income distribution among Sari-Sari store owners in the greater Manilla (Philippines) area (?-distributed)
    - **Estimating the distribution of Y could be an essential component of the study itself** to the extent that it either provides a better sense of a novel or unfamiliar phenomena, informs subsequent design, or enables researchers to better incorporate information on the error distribution into subsequent analyses
  - When designing studies which involve regionally-situated phenomena, have to recognize that “social” space and “social” geographies do not necessarily align with administrative space/geographies

Common Tacts

- Conventional statistical analyses are parametric, require researchers to <u>specify the distribution of the outcome Y</u>
  - Consequences of **misspecification** (bias, incorrect SEs / CIs, mis-interpretation of effects) can be substantial (e.g., Long, 1997), and not necessarily remedied by just “getting a larger sample”

- Common solutions:
  - **Transform Y** to better fit a conventional model (i.e., ln(Y), Y^{-1})
  - **GLMs** (McCullagh and Nelder, 1989) to model Y on it’s native scale, obtain correct SEs, reasonable-to-interpret effects (i.e., gamma regression, negative binomial regression, etc.)
    - However, as with transformations, <u>GLMs not allow differential-modeling of Y</u> distributions (can’t transform some groups but not others)

QUANTILE REGRESSION AND BAYESIAN QUANTILE REGRESSION

Quantile Regression

- Quantile regression (QR) offers a reasonable and highly flexible (more comprehensive) alternative to GLMs in many scenarios (Davino et al., 2014; Koenker, 2005)
  - GLMs: model conditional mean
  - QR: models conditional quantiles (median, 75^{th} percentile, etc.)

- OLS vs. QR loss functions
  - OLS: $\mu = \text{argmin}_c\ E[Y\text{-}c]^2$
  - QR for Me: $Me = \text{argmin}_c\ E|Y\text{-}c|$
  - QR for general quantile $\theta = P(Y \leq y)$: $q_\theta = \text{argmin}_c\ E|\rho_\theta(Y\text{-}c)|$

The ***ρ*** *are often referred to as* ***weights****, which are defined by a* ***check function***

$$\rho_\theta = [(1\text{-}\theta)\ I(y \leq 0)] + \theta\ I(y>0)]\ |y|$$

  - QR conditional on X for quantile θ: $b(\theta) = \text{argmin}_b\ E|\rho_\theta(Y\text{-}Xb)|$

Quantile Regression - II

- In the contexts described above, QR can be particularly attractive option (Davino et al., 2014; Koenker, 2005; McMillen, 2013)
  - **Inferences about Y | X relationships are distribution free**: method makes no assumption about the distribution of Y
    - Put differently, QR makes no assumptions about the error distribution for Y, and is thus robust to model misspecification (QR can in fact be used to **estimate the distribution of Y | X**)
  - QR readily amenable to **estimating percentile intervals in the data** (i.e., "80% of white male respondents with 16 years of education have incomes within the range $Y_L - Y_U$")
    - Potentially useful property for clients who seek an alternate way of understanding where substantively meaningful slices of the population fall
    - Relatedly, QR is amenable to **threshold analyses**

Going Bayesian

- Classical QR is not new per se; Bayesian QR is (relatively)
- In general, Bayesian methods allow researchers to model not just response Y, but also regression coefficients (Carlin and Louis, 2009):

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_i + e$$

$$\beta_{0j} = \gamma_{00} + \mu_{0j}$$

$$\beta_{1j} = \gamma_{10} + \mu_{1j}$$

- Treating Y and β_p as explicit random variables has numerous advantages in research where **contexts (geographic, social) are important** (Gelman and Hill, 2006)
  - **β_p can vary across aggregations** (such as regions or socio-political entities) in a way that adjusts for (allows evaluation of) contextual / cluster differences (Raudenbush and Bryk, 2001)
    - **Example from education research**: what are the characteristics of schools which have lower black/white test score differentials? How are school means correlated with black/white differentials?

Going Bayesian - II

- Bayesian paradigm provides principled mechanisms for **incorporating results from previous studies** or data collection efforts into analysis
  - Elicited priors (Gill and Walker, 2005; see also Rendell et al., 2009)
  - Bayesian updating for longitudinal studies (Carlin and Louis, 2009)
  - **WIPs** to stabilize estimation (Chung et al., 2015; Gelman et al., 2008)
- Suggests that payoffs to <u>integrating Bayesian methods with QR approach</u> could be substantial …

Bayesian Quantile Regression

- **Bayesan quantile regression (BQR)** attempts to achieve the same flexibility that HLMs / HGLMs enjoy – specification of priors for model coefficients, contextualizing models via random effects
  - Also, robustify quantile-specific estimates with **small samples**

- Bayesian extensions of QR have a couple of different flavors:
  - **Associated likelihood is approximate** and there are a handful of different ways to parameterize (e.g., Feng, 2015; Yang and He, 2012; Yu et al., 2001), some of which are slightly more robust than others in certain contexts
    - And not all of which allow joint estimation of quantiles…

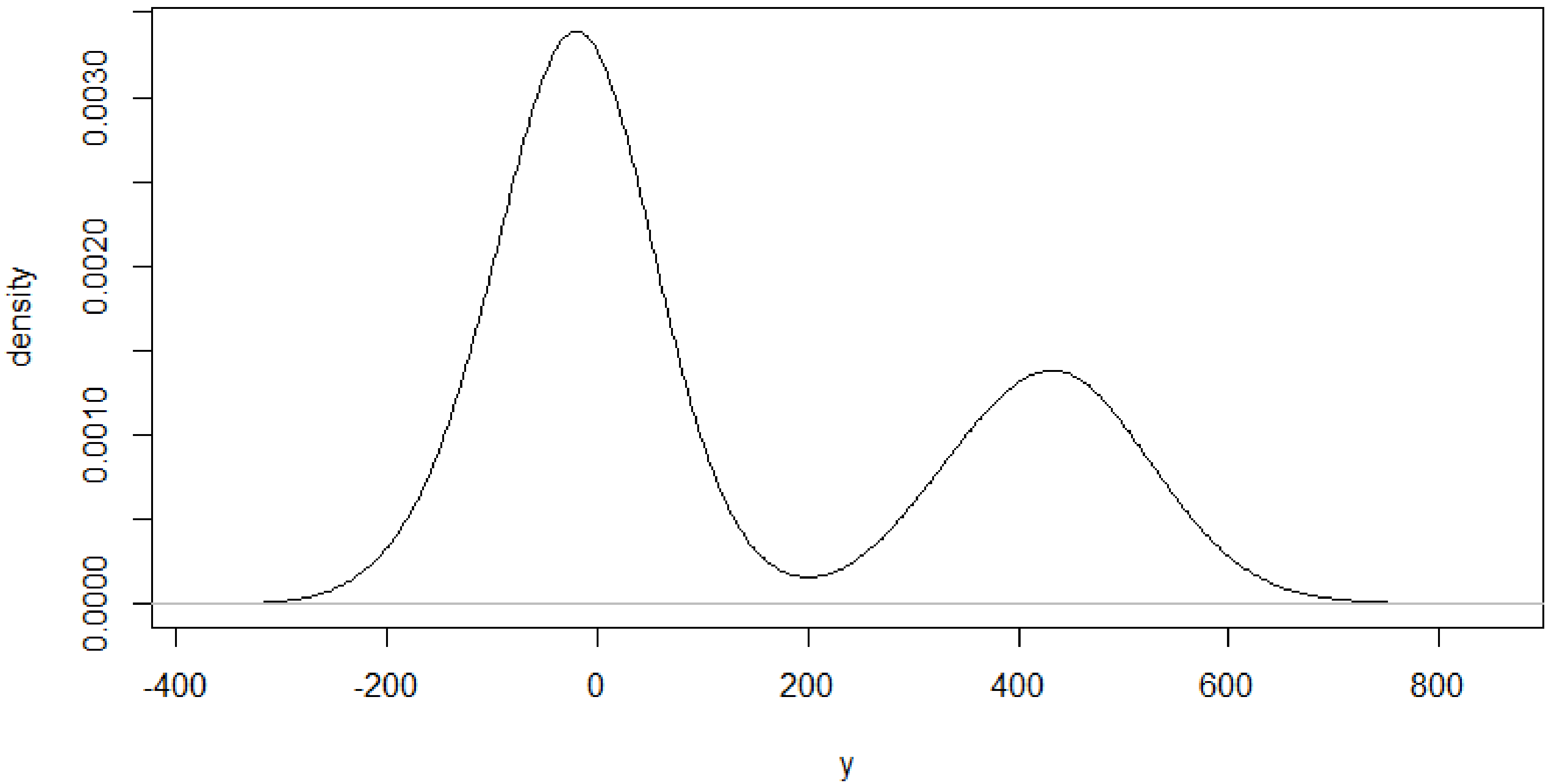
- Still, another payoff of to Bayesifying QR is that can potentially integrate **Bayesian approaches to incorporating sample design information** (Gelman et al., 2013; Si et al., 2015)

ILLUSTRATION

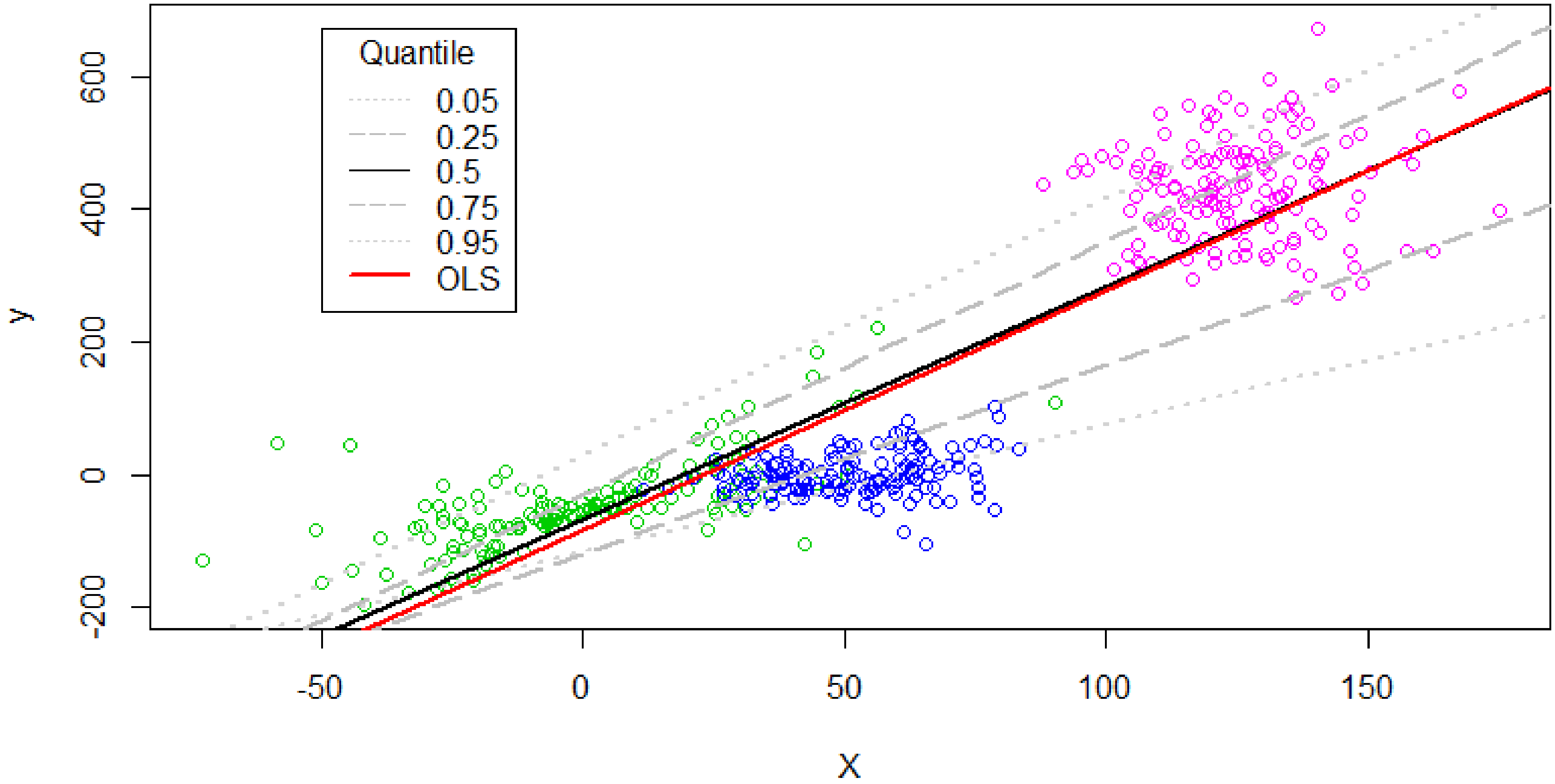
Simulation

- Data were generated from three different population distributions, N=10,000
  - Normal(0,25)
  - Gamma(10,5)
  - Chi Squared(125)
- A relationship with the response variable was applied (Y|X), and the data were combined to form a dependent variable with a single continuous independent variable
- Sample draws made of n=150 per distribution
- The error is heterosckedastic, non-normal, and differing distributions across the values of the independent variable
  - In other words, a worst case scenario for performing OLS

The Density of the Response



Best-Fit Lines (OLS, BQR with plat prior)



Why BQR vs. OLS?

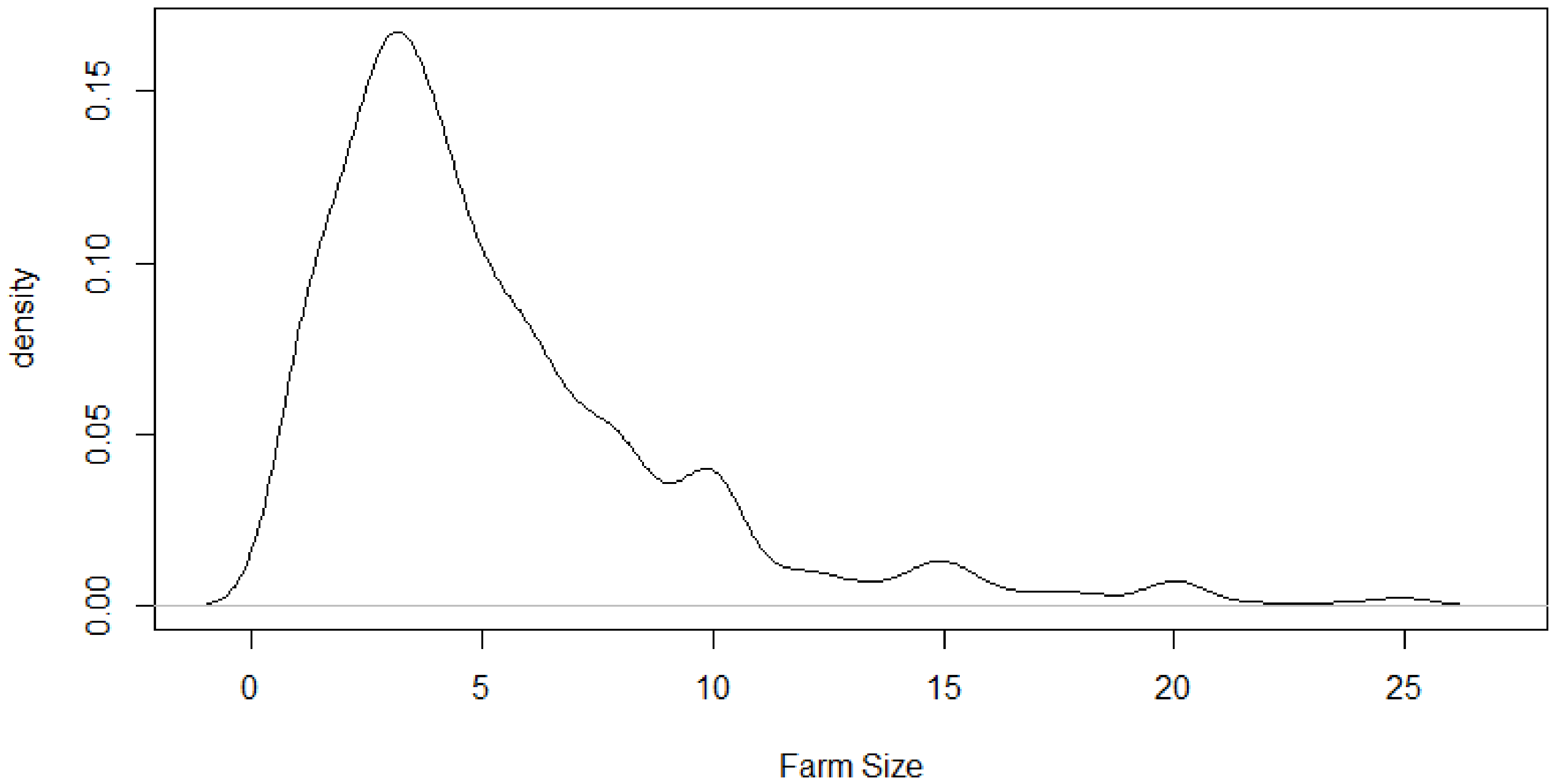
- The OLS line and the median regression line are very close, in spite of the asymmetry of Y … so why use quantile regression?

  - **It is impossible to get accurate estimates of the standard error for the OLS slope** without a significant increase in the complexity of the model (and would model interpretability be compromised?)
    - I.e., even though the β estimates are aligned (parametric vs. nonparametric), SEs for the OLS line are incorrect because the distribution of Y is not correctly specified (QR does not depend upon proper specification of Y, and so SEs are more robust)

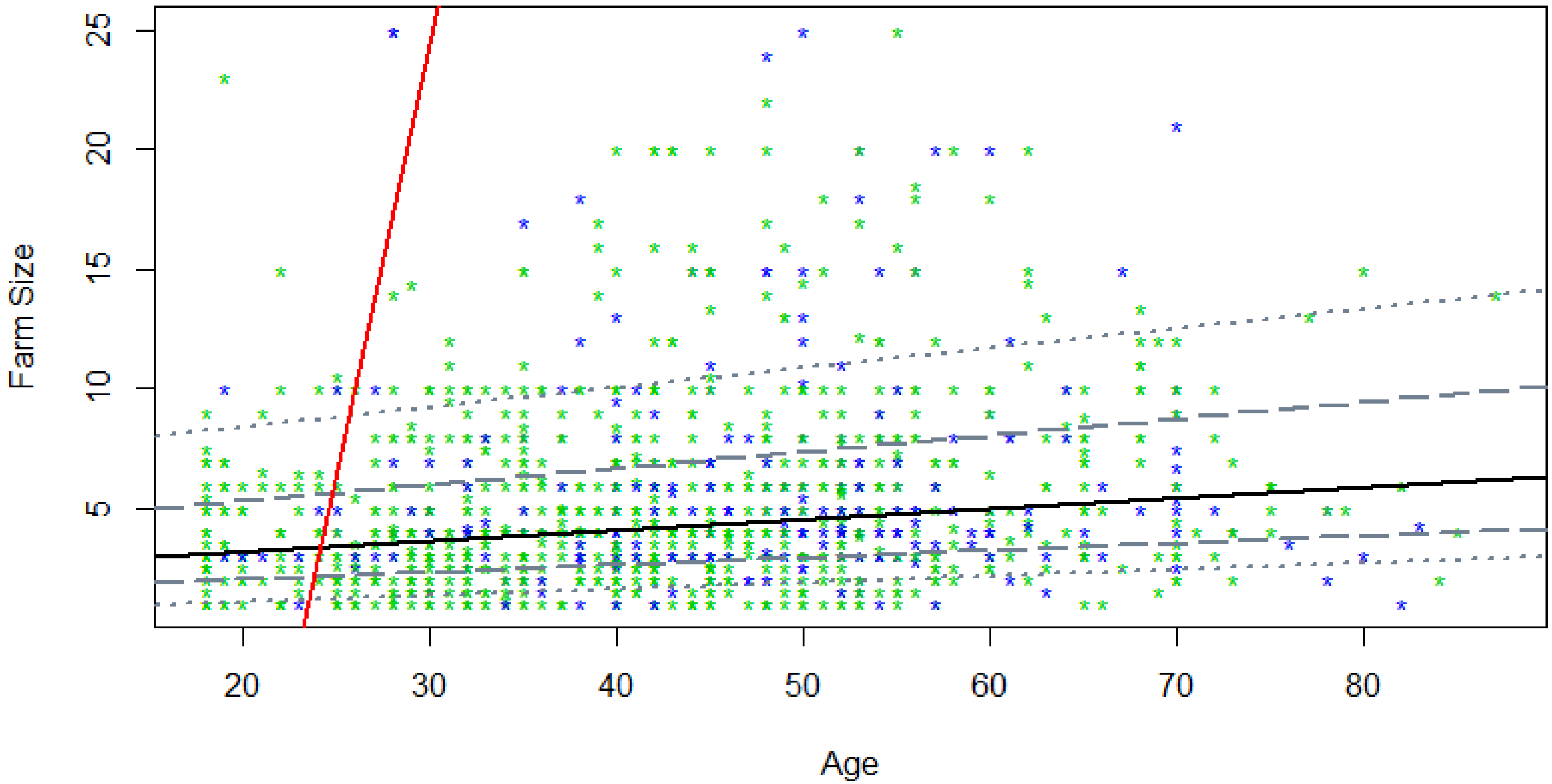
  - We can **reliably understand the spread of the response** with quantile regression because the distance between quantile lines is a measure of spread, e.g. interquartile range, θ differences, etc.

  - In this case, **flat prior means that BQR result is same as QR result**
    - Subsequent analyses show greater stability in upper/lower quantiles if alter informativeness

Actual Agricultural Example: Density of Y



Best-Fit Lines (OLS, BQR), Untrimmed Y



CONCLUSIONS AND NEXT STEPS

Concluding Thoughts

- QR is well-suited to a number of problems that arise in comparative survey research
  - BQR extensions have potential to allow flexible incorporation of survey information
- Additional simulations show that QR/BQR is reasonably robust to common model misspecifications
  - Error, X correlation, failure to include true region information
- In spite of this, full BQR is not as plug-and-play as, say, HLM
  - Conscientious implementation requires careful consideration of approximated likelihood, and reflection on prior (basic eBayes not as intuitive)

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CITATIONS AND REFERENCES

Citations and References

- Cai, T. 2013. "Investigation of Ways to Handle Sampling Weights for Multilevel Model Analyses" *Sociological Methodology* 43: 178-219.
- Carlin, B. P., and Louis, T. A. 2009. Bayesian Methods for Data Analysis. Third Edition. CRC Press.
- Congdon, P. 2005. Bayesian Models for Categorical Data. Wiley.
- Davino, C., Furno, M., and Vistocco, D. 2014. Quantile Regression: Theory and Applications. Wiley.
- Feng, Y., Chen, Y., and He, X. 2015. "Bayesian Quantile Regression with Approximate Likelihood" *Bernoulli* 21: 832-850.
- Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., and Rubin, D. B. 2013. Bayesian Data Analysis. Chapman and Hall/CRC.
- Gelman, A., Jakulin, A., Pittau, M. G., and Su, Y.-S. 2008. "A Weakly Informative Default Prior Distribution for Logistic and Other Regression Models" *Annals of Applied Statistics* 4: 1360 – 1383.
- Gelman, A., and Hill, J. 2006. Data Analysis Using Regression and Multilevel/Hierarchical Models. Cambridge.
- Gill, J. and Walker, L. D. 2005. "Elicited Priors for Bayesian Model Specifications in Political Scienc Research" *Journal of Politics* 67: 841-872.
- Koenker, R. 2005. Quantile Regression. Cambridge.
- Long, J. S. 1997. Regression Models for Categorical and Limited Dependent Variables. Sage.

Citations and References - II

- McCullagh, P., and Nelder, J. A. 1989. Generalized Liner Models. Second Edition. Chapman and Hall / CRC.
- McMillen, D. P. 2013. Quantile Regression for Spatial Data. Springer.
- Raudenbush, S. W., and Bryk, A. S. 2001. Hierarchical Linear Models. Sage.
- Rendell, M.S., Handcock, M. S., and Jonsson, S. H. 2009. “Bayesian Estimation of Hispanic Fertility Hazards from Survey and Population Data” *Demography*. 46: 65-83.
- Reich, B. J., Bondell, H. D., and Wang, H. 2010. “Flexible Bayesian Quantile Regression for Independent and Clustered Data” *Biostatistics* 11: 337-352.
- Si, Y., Pillai, N., and Gelman, A. 2015. “Bayesisan Nonparametric Weighted Sampling Inference” *Bayesian Analysis*. 1: 1-21.
- Yang, Y. and He, X. 2012. “Bayesian Empirical Likelihood for Quantile Regression” *Annals of Statistics* 40: 1102 – 1131.
- Yu, K., and Moyeed, R. A. 2001. “Bayesian Quantile Regression” *Statistics and Probability Letters*. 54: 437-447.

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