Quantile Regression as a Tool for Cross-National and Comparative Survey Research
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CONTEXT AND MOTIVATING CHALLENGES
Qualitative and Quantitative Variation

• Survey and evaluation research require researchers to take into account the particulars of populations and phenomena studied.

• In agricultural surveys, can often see qualitative variation in key measures across regions and countries, especially when subset on key groups.
  – Distribution of arable soil can impact crop yields and crop quality dramatically across study regions.

• In longitudinal evaluation research, it is reasonable to expect that the intervention itself can alter both the qualitative and quantitative nature of the focal measures.
  – A business intervention which promotes shifting product mixes to enhance profitability could lead to temporary re-alignment of goods and services sold and a thus temporary or longer-term shifts in the distribution of sales / profits.
  – An intervention which promotes financial record keeping could lead to qualitative shifts in the distributions of reported income.
Quantitative vs. Qualitative Variation

Regime 1

Regime 2

Regime 3

All Regimes, Pooled
Other Considerations

• When setting up (parameterizing) quantitative analyses, have to be context-sensitive and not impose prior conceptions on what population phenomena look like

• US overall income distribution (gamma-distributed) vs. income distribution among Sari-Sari store owners in the greater Manilla (Philippines) area (?-distributed)

  — Estimating the distribution of Y could be an essential component of the study itself to the extent that it either provides a better sense of a novel or unfamiliar phenomena, informs subsequent design, or enables researchers to better incorporate information on the error distribution into subsequent analyses

• When designing studies which involve regionally-situated phenomena, have to recognize that “social” space and “social” geographies do not necessarily align with administrative space/geographies
Common Tacts

• Conventional statistical analyses are parametric, require researchers to specify the distribution of the outcome Y
  • Consequences of misspecification (bias, incorrect SEs / CIs, mis-interpretation of effects) can be substantial (e.g., Long, 1997), and not necessarily remedied by just “getting a larger sample”

• Common solutions:
  • Transform Y to better fit a conventional model (i.e., ln(Y), Y^{-1})
  • GLMs (McCullagh and Nelder, 1989) to model Y on it’s native scale, obtain correct SEs, reasonable-to-interpret effects (i.e., gamma regression, negative binomial regression, etc.)

  ‒ However, as with transformations, GLMs not allow differential-modeling of Y distributions (can’t transform some groups but not others)
QUANTILE REGRESSION AND BAYESIAN QUANTILE REGRESSION
Quantile Regression

• Quantile regression (QR) offers a reasonable and highly flexible (more comprehensive) alternative to GLMs in many scenarios (Davino et al., 2014; Koenker, 2005)
  • GLMs: model conditional mean
  • QR: models conditional quantiles (median, 75th percentile, etc.)

• OLS vs. QR loss functions
  • OLS: \[ \mu = \arg\min_c E[(Y-c)^2] \]
  • QR for Me: \[ \text{Me} = \arg\min_c E|Y-c| \]
  • QR for general quantile \( \theta = P(Y \leq y) \): \[ q_\theta = \arg\min_c E|\rho_\theta(Y-c)| \]
  • QR conditional on \( X \) for quantile \( \theta \): \[ b(\theta) = \arg\min_b E|\rho_\theta(Y-Xb)| \]

\( \rho \) are often referred to as weights, which are defined by a check function.
Quantile Regression - II

• In the contexts described above, QR can be particularly attractive option (Davino et al., 2014; Koenker, 2005; McMillen, 2013)

• Inferences about $Y \mid X$ relationships are distribution free: method makes no assumption about the distribution of $Y$

  – Put differently, QR makes no assumptions about the error distribution for $Y$, and is thus robust to model misspecification (QR can in fact be used to estimate the distribution of $Y \mid X$)

• QR readily amenable to estimating percentile intervals in the data (i.e., “80% of white male respondents with 16 years of education have incomes within the range $Y_L – Y_U$”)

  – Potentially useful property for clients who seek an alternate way of understanding where substantively meaningful slices of the population fall

  – Relatedly, QR is amenable to threshold analyses
Going Bayesian

• Classical QR is not new per se; Bayesian QR is (relatively)
• In general, Bayesian methods allow researchers to model not just response Y, but also regression coefficients (Carlin and Louis, 2009):

\[ Y_{ij} = \beta_{0j} + \beta_{1j}X_i + e \]

\[ \beta_{0j} = \gamma_{00} + \mu_{0j} \]
\[ \beta_{1j} = \gamma_{10} + \mu_{1j} \]

• Treating Y and \( \beta_p \) as explicit random variables has numerous advantages in research where contexts (geographic, social) are important (Gelman and Hill, 2006)

• \( \beta_p \) can vary across aggregations (such as regions or socio-political entities) in a way that adjusts for (allows evaluation of) contextual / cluster differences (Raudenbush and Bryk, 2001)

  – Example from education research: what are the characteristics of schools which have lower black/white test score differentials? How are school means correlated with black/white differentials?
Going Bayesian - II

• Bayesian paradigm provides principled mechanisms for **incorporating results from previous studies** or data collection efforts into analysis
  – Elicited priors (Gill and Walker, 2005; see also Rendell et al., 2009)
  – Bayesian updating for longitudinal studies (Carlin and Louis, 2009)
  – **WIPs** to stabilize estimation (Chung et al., 2015; Gelman et al., 2008)

• Suggests that payoffs to **integrating Bayesian methods with QR approach** could be substantial …
Bayesian Quantile Regression

- **Bayesian quantile regression (BQR)** attempts to achieve the same flexibility that HLMs / HGLMs enjoy – specification of priors for model coefficients, contextualizing models via random effects
  - Also, robustify quantile-specific estimates with **small samples**

- Bayesian extensions of QR have a couple of different flavors:
  - **Associated likelihood is approximate** and there are a handful of different ways to parameterize (e.g., Feng, 2015; Yang and He, 2012; Yu et al., 2001), some of which are slightly more robust than others in certain contexts
    - And not all of which allow joint estimation of quantiles...

- Still, another payoff of to Bayesifying QR is that can potentially integrate **Bayesian approaches to incorporating sample design information** (Gelman et al., 2013; Si et al., 2015)
ILLUSTRATION
Simulation

• Data were generated from three different population distributions, N=10,000
  – Normal(0,25)
  – Gamma(10,5)
  – Chi Squared(125)

• A relationship with the response variable was applied (Y|X), and the data were combined to form a dependent variable with a single continuous independent variable

• Sample draws made of n=150 per distribution

• The error is heteroscedastic, non-normal, and differing distributions across the values of the independent variable
  • In other words, a worst case scenario for performing OLS
The Density of the Response
Best-Fit Lines (OLS, BQR with plat prior)
Why BQR vs. OLS?

• The OLS line and the median regression line are very close, in spite of the asymmetry of Y ... so why use quantile regression?

• **It is impossible to get accurate estimates of the standard error for the OLS slope** without a significant increase in the complexity of the model (and would model interpretability be compromised?)
  
  – I.e., even though the $\beta$ estimates are aligned (parametric vs. nonparametric), SEs for the OLS line are incorrect because the distribution of Y is not correctly specified (QR does not depend upon proper specification of Y, and so SEs are more robust)

• We can **reliably understand the spread of the response** with quantile regression because the distance between quantile lines is a measure of spread, e.g. interquartile range, $\theta$ differences, etc.

• In this case, **flat prior means that BQR result is same as QR result**
  
  – Subsequent analyses show greater stability in upper/lower quantiles if alter informativeness
Actual Agricultural Example: Density of Y
Best-Fit Lines (OLS, BQR), Untrimmed Y
CONCLUSIONS AND NEXT STEPS
Concluding Thoughts

- QR is well-suited to a number of problems that arise in comparative survey research
  - BQR extensions have potential to allow flexible incorporation of survey information

- Additional simulations show that QR/BQR is reasonably robust to common model misspecifications
  - Error, X correlation, failure to include true region information

- In spite of this, full BQR is not as plug-and-play as, say, HLM
  - Conscientious implementation requires careful consideration of approximated likelihood, and reflection on prior (basic eBayes not as intuitive)
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